IMPROVED ROTORDYNAMIC UNBALANCE RESPONSE CALCULATIONS USING THE POLYNOMIAL METHOD

Brian T. Murphy
Rotordynamics-Seal Research, Inc.
Simi Valley, CA

ABSTRACT

For rotordynamic calculations the conventional transfer matrix (CTM) method is known for its ease of application and speed of execution. This is true for eigenvalue analyses as well as forced response calculations. In 1983, Murphy and Vance significantly improved the conventional transfer matrix method for eigenvalues by developing Polynomial Transfer Matrix (PTM) method. For eigenvalue calculations this resulted in one to two orders of magnitude increase in calculation speed, and eigenvalues could be found without missing any modes. This paper extends the polynomial method to forced response calculations. The polynomial method "compiles" a polynomial representation of a rotor bearing system. Responses at many frequency increments can then be computed with amazing rapidity. Similar to the case of eigenvalue calculations, an automatic dynamic condensation feature helps the PTM attain nearly a tenfold increase in speed over the CTM method. This paper details the polynomial method applied to forced response calculations. Two example calculations are presented to compare the PTM to the CTM method and finite element method. All computations were carried out on an IBM-PC compatible computer.

NOMENCLATURE

A	beam cross sectional area
CTM	Conventional Transfer Matrix
E,G	Young's modulus and shear modulus
$\{E\}$	overall system excitation vector
f	real valued scale factor
$\{F\}$	unbalance excitation vector for one station
FE	Finite Element
i	$\sqrt{-1}$
I	second moments of area for beam bending
L	station beam length
K,C	bearing stiffness and damping values, direct and cross-coupled
m_θ,m_ϕ	real valued beam bending moments
M_θ,M_ϕ	complex valued beam bending moments
$[M]$	mass transfer matrix for one station
N	number of stations in the overall model
PTM	Polynomial Transfer Matrix
q	state vector of transfer variables defined in eq. (5a)
s	complex speed variable equal to $i\omega$
S	scaled speed variable defined in eq. (10)
$[S]$	shaft transfer matrix for one station
t	time
$[T]$	overall system transfer matrix
u_x,u_y	real valued unbalance magnitude in units like gram-inches
U_x,U_y	complex valued unbalance magnitude in units like gram-inches
v_x,v_y	real valued beam shear
V_x,V_y	complex valued beam shear
w,T,P	station mass, transverse inertia and polar inertia
x,y	real valued translational inertial coordinates
X,Y	complex valued translational inertial coordinates
α	shear shape factor for shear deflection term
β	unbalance circumferential location in radians
θ,ϕ	real valued rotational inertial coordinates
Θ,Φ	complex valued rotational inertial coordinates
ω	shaft speed in radians per second

*	superscript to denote complex conjugate
'	superscript to denote state variable to right of mass
n	subscript to denote an individual station

INTRODUCTION

The Transfer Matrix Method has long been a viable way to analyze the dynamics of certain classes of machines. When Holzer (1921) introduced the method for torsional natural frequency calculation for shafts, he began by showing how a complex machine could be modeled as a series of discrete lumps, or elements. The equations of motion for each lump were written separately. Taken together, all the equations for all the lumps form a system of equations from which natural frequencies and mode shapes could be obtained (i.e., matrix equation). Before the age of the digital computer, efficiency of calculation was of utmost importance. Eigenvalues and eigenvectors for large order matrix problems could not be readily computed with just a slide rule. So Holzer arranged the equations for each lump to form a transfer equation. He showed that by using transfer equations, one could readily compute undamped natural frequencies and mode shapes for a restricted class of multi-degree-of-freedom systems with relatively little calculation effort.

In retrospect, Holzer had devised an extremely efficient eigenvalue extraction algorithm for purely tridiagonal matrices. Tridiagonal matrices have nonzero elements only on the main diagonal, and the diagonals immediately above and below the main diagonal. Torsional (or axial) vibration analysis of beams and shafts lead to a tridiagonal system matrix. Holzer's method is also efficient at computing response amplitudes to harmonic forcing functions, or to statically applied loads. Since Holzer's time many published articles have strived to extend Holzer's method, in one way or another, beyond the purely tridiagonal limitation. In the field of machine dynamics, the most important extension was made concurrently by Myklestad (1944) and Prohl (1945) to the lateral vibration analysis of airplane wings and rotating shafts, respectively. Whereas a beam torsion model has one degree-of-freedom per lump, beam bending requires two degrees-of-freedom per lump. Each element of Holzer's tridiagonal matrix was now a two by two square submatrix.

Subsequently Lund and Orcutt (1967) and Lund (1974), in the field of rotordynamics, applied the transfer methodology to the case of cross-coupled beam bending, or four degrees-of-freedom per lump with four by four square submatrices. Because of its combination of power, speed and ease of programming, the conventional transfer matrix (CTM) method became the method of choice for rotordynamic computer programs, both for eigenanalysis and response. This happened in spite of known difficulties in using the CTM method to a) iteratively search for damped eigenvalues, and b) compute higher order modes. In most practical cases b) is not a significant limitation, and in any event is adequately dealt with by using the Ricatti transfer matrix technique (see Horner, et al, 1977). Situation a) is potentially more severe since it can lead to missed modes during calculation, especially rotordynamically unstable modes. Murphy and Vance (1983) eliminated this problem when they introduced the polynomial method. They retained all the features of Lund's formulation, but carried out the calculations in a new way to compute the system characteristic polynomial. In return, they effectively eliminated the problem of missed modes, and also gained one to two orders of magnitude in execution speed. A large part of this speed gain was due to a dynamic condensation technique made possible by the polynomial approach (Murphy (1984) or Vance (1988)). The transfer matrix method now worked much better and ran much faster with the use of polynomials.

The original work by Murphy and Vance focused on eigenvalue analysis. Applying the polynomial transfer matrix (PTM) method to response calculations has not been attempted until now. This was partly because it was believed by this author that the Finite Element (FE) method would displace the transfer matrix method for all rotordynamic calculations. This, however, has not occurred primarily because the PTM method for eigenanalysis is typically several orders of magnitude faster than any FE method program running on the same computer platform. This difference simply cannot be ignored especially when low cost computers (i.e., IBM-PC's) are used to perform an ever expanding share of engineering calculations. The FE method is more general and is equally applicable to arbitrarily complex models as it is to limited models like the transfer matrix method. Again, however, the additional complexity and much slower execution has prevented it from displacing the transfer matrix method for most rotordynamic calculations.

In this article it will be shown how the polynomial method for transfer matrices can be used to compute harmonic response for rotating shafts. This encompasses unbalance response, operating deflected shapes, and response to arbitrary harmonic excitations like sweeping blade and vane pass frequencies. It will also be shown how automatic dynamic condensation can be used to greatly speed up the calculations with no loss of accuracy. Two example cases, a uniform beam and multi-stage compressor, will be used to demonstrate the speed and accuracy of the method. Direct comparisons will be presented versus both CTM and FE analysis programs.

ANALYSIS

The basic transfer matrix formulae presented here for rotor/bearing systems follow directly from Lund (1974). To begin, the shaft is modeled as a series of stations, Figure 1. Each station consist of a rigid cylindrical disk and a length of massless shaft, Figure 2. The dynamical equations of motion (F=ma) are written for the disk and arranged into transfer matrix form (see Lund (1974) for a more thorough discussion).

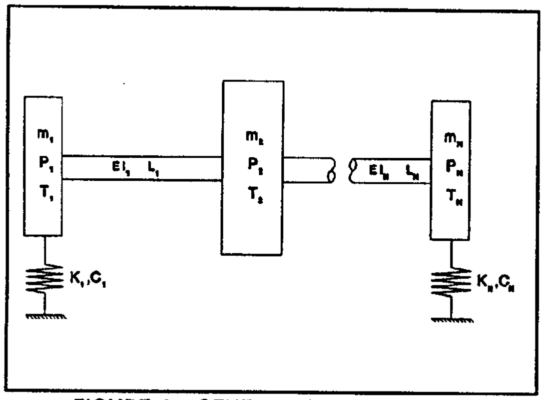

FIGURE 1. GENERAL SHAFT MODEL.

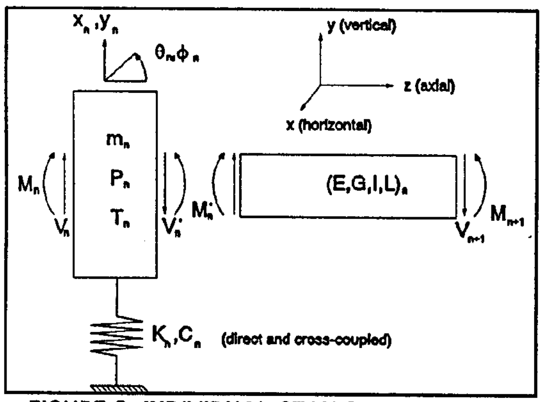

FIGURE 2. INDIVIDUAL STATION DEFINITION.

$$
\begin{aligned}
X_n' &= X_n \\
Y_n' &= Y_n \\
\Theta_n' &= \Theta_n \\
\Phi_n' &= \Phi_n \\
-V_{xn}' &= -V_{xn} + s^2 w_n X_n + K_{xx} X_n + K_{xy} Y_n + sC_{xx} X_n + sC_{xy} Y_n - U_{xn} \\
-V_{yn}' &= -V_{yn} + s^2 w_n Y_n + K_{yx} X_n + K_{yy} Y_n + sC_{yx} X_n + sC_{yy} Y_n - U_{yn} \\
M_{\theta n}' &= M_{\theta n} + s^2 T_n \Theta_n - is^2 P_n \Phi_n + K_{\theta\theta}\Theta_n + K_{\theta\phi}\Phi_n + sC_{\theta\theta}\Theta_n + \\
&\qquad sC_{\theta\phi}\Phi_n - U_{\theta n} \\
M_{\phi n}' &= M_{\phi n} + s^2 T_n \Phi_n + is^2 P_n \Theta_n + K_{\phi\theta}\Theta_n + K_{\phi\phi}\Phi_n + sC_{\phi\theta}\Theta_n + \\
&\qquad sC_{\phi\phi}\Phi_n - U_{\phi n}
\end{aligned}
\tag{1a-h}
$$

These formulae include the effects of disk gyroscopics, linear bearings to ground, and a harmonic forcing function U of frequency ω (or $-is$). Note the use of complex notation and the assumed solution form of

$$x(t) = x_c \cos\omega t + x_s \sin\omega t = ½\left(Xe^{st} + X^* e^{s^* t}\right) \tag{2a}$$

$$y(t) = y_c \cos\omega t + y_s \sin\omega t = ½\left(Ye^{st} + Y^* e^{s^* t}\right) \tag{2b}$$

where

$$X = x_c - ix_s$$
$$Y = y_c - iy_s$$
$$s = i\omega$$

and similarly for Θ, Φ, V_x, V_y, M_θ, M_ϕ.

When the forcing function is rotating unbalance, it takes the form

$$u_x(t) = u\omega^2\cos(\omega t+\beta) = ½\left(-us^2e^{(st+i\beta)} - us^2e^{(s^*t-i\beta)}\right) \tag{3a}$$
so $U_{xn} = -ue^{i\beta}s^2$

$$u_y(t) = u\omega^2\sin(\omega t+\beta) = ½(-i)\left(-us^2e^{(st+i\beta)} - us^2e^{(s^*t-i\beta)}\right) \tag{3b}$$
so $U_{yn} = iue^{i\beta}s^2$

where

- u = unbalance amount (i.e., in-lb, oz-in, gm-in, etc.)
- β = unbalance circumferential location (+ in direction of rotation)
- $s^2 = i^2\omega^2 = -\omega^2$
- $U_\theta = 0$
- $U_\phi = 0$

Equations (1), (2) and (3) show how complex notation can be used to solve the problem, and how this relates directly back to the real solution in terms of real valued trigonometric functions. When carrying out actual calculations employing complex arithmetic, only the first terms on the right sides of equations (2) and (3) need be operated on. The solution due to the conjugate terms is then simply the conjugate of the solution already computed (a result of the equations (1) being linear).

Next, static deflection equations for the massless length of shaft are written using Euler beam and Timoshenko shear formulae. These expressions are arranged in transfer matrix form resulting in (see Lund (1974) for additional terms to include hysteretic internal shaft damping):

$$
\begin{aligned}
X_{n+1} &= X_n + L_n\Theta_n + \frac{1}{(EI)_n}\left[\frac{L_n^2}{2}M'_{\theta n} + \left(\frac{L_n^3}{6} - \frac{(LEI)_n}{(\alpha GA)_n}\right)V'_{xn}\right] \\
Y_{n+1} &= Y_n + L_n\Phi_n + \frac{1}{(EI)_n}\left[\frac{L_n^2}{2}M'_{\phi n} + \left(\frac{L_n^3}{6} - \frac{(LEI)_n}{(\alpha GA)_n}\right)V'_{yn}\right] \\
\Theta_{n+1} &= \Theta_n + \frac{1}{(EI)_n}\left[L_n M'_{\theta n} + \frac{L_n^2}{2}V'_{xn}\right] \\
\Phi_{n+1} &= \Phi_n + \frac{1}{(EI)_n}\left[L_n M'_{\phi n} + \frac{L_n^2}{2}V'_{yn}\right] \\
M_{\theta,n+1} &= M'_{\theta n} + L_n V'_{xn} \\
M_{\Phi,n+1} &= M'_{\Phi n} + L_n V'_{yn} \\
V_{x,n+1} &= V'_{xn} \\
V_{y,n+1} &= V'_{yn}
\end{aligned} \tag{4}
$$

In matrix notation equations (1) and (4) can be written as

$$q'_n = [M]_n q_n + F_n \tag{5a}$$

$$q_{n+1} = [S]_n q'_n \tag{5b}$$

Equation (5a) can be substituted into equation (5b) to eliminate q'_n as follows:

$$q_{n+1} = [S]_n [M]_n q_n + [S]_n F_n \tag{6}$$

Equations (5a) and (5b) are the transfer equations for the lumped mass, and shaft section, respectively. Equation (6) is the transfer equation for the entire station. The matrices $[S]$ and $[M]$ are eight by eight. The vectors q and F are eight by one. Repeated application of equation (6) produces the transfer equation for the entire shaft system as follows:

$$q_N = [M]_N [S]_{N-1} [M]_{N-1} \cdots [S]_1 [M]_1 q_1 + \sum_{n=1}^{N} ([M]_N [S]_{N-1} [M]_{N-1} \cdots [S]_n) F_n \tag{7a}$$

$$q_N = [T] q_1 + E \tag{7b}$$

The matrix notation in equation (7b) represents eight scaler equations. The system transfer matrix is $[T]$, and E is the system excitation vector. By taking the bottom four equations and inserting the boundary conditions of zero shear and zero bending moment at the shaft ends, one can solve for the shaft deflections at station number one.

$$0 = [T_{LL}] \begin{Bmatrix} x \\ y \\ \theta \\ \phi \end{Bmatrix}_1 + E_L \tag{8}$$

where

$[T_{LL}]$ = lower left four by four submatrix of T
E_L = lower four by one subvector of E

After solving equation (8) for $\{xy\theta\phi\}_1$, the deflections at all remaining stations are obtained by repeated use of equation (6).

In the CTM method the speed variable s is assigned a numerical value, and the matrix calculations of equation (7a) are computed to produce equation (8). Equation (8) is solved for $\{xy\theta\phi\}_1$, and equation (6) is used to complete the solution. All these calculations are carried out in entirety for every speed increment. This is actually a very efficient procedure for solving a system of 4N equations for 4N unknowns, and is far more efficient on both computer time and memory than, say, Gaussian elimination on a 4N by 4N square system matrix. In the PTM method the speed variable is not assigned a numerical value, but actually remains a variable in the matrix multiplications of equation (7a). This is facilitated by the fact that all elements of $[M]_n$, $[S]_n$ and F_n are simple polynomials in s. The elements of $[S]_n$ are zero degree polynomials, and the elements of $[M]_n$ and F_n are two degree polynomials in s. By using straightforward polynomial multiplications the elements of $[T]$ and E in equation (7b) will be polynomials of degree $2N$ and $2N+2$, respectively. It is at this point that a numerical value for s is selected and substituted to produce equation (8), and the remaining steps in the procedure are identical to the CTM method.

It takes more effort to compute the polynomial form of equation (7a) than to make a single pass through equation (7a) with a numerical value for s. However, the numerical effort of evaluating the polynomials for different s values is very small. This means that if enough speed increments are done, the PTM method will be faster. Other than that, the CTM and PTM methods will produce identical answers for responses. This will be demonstrated in the following sections, along with the effect of automatic dynamic condensation.

AUTOMATIC DYNAMIC CONDENSATION

The technique of automatic dynamic condensation was developed by Murphy (1983). It forms an important part of the PTM method for both eigenvalue and response analysis by speeding up the calculations by typically more than a factor of ten. Unlike other condensation techniques common to the FE method, polynomial condensation is extremely easy to apply, and when properly implemented there is absolutely no measurable loss of accuracy.

For both the CTM and PTM methods parameter scaling is necessary to avoid numerical overflow or underflow. For example, on an IBM-PC the largest and smallest allowable numbers are about 1E±307 in double precision for most compilers. Without scaling, the coefficients of the polynomials becomes prohibitively small. Consider the following n degree polynomial:

$$P = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + ... + a_n s^n \qquad (9)$$

The overall trend for rotordynamic analysis is the larger the degree of the term, the smaller the coefficient. In other words, for the coefficient a_i, as i increases the magnitude of a_i decreases. When n is large due to a large number of stations used, the value of a_i for i approaching n will be smaller than the allowable limit 1E-307. The most convenient way to combat this problem is to make a simple substitution into equation (9).

$$s = fS \qquad (10)$$

where f is termed the scale factor. The scaled polynomial is then

$$\begin{aligned} P &= a_0 + a_1 fS + a_2 f^2 S^2 + a_3 f^3 S^3 + ... + a_n f^n S^n \\ &= A_0 + A_1 S + A_2 S^2 + A_3 S^3 + ... + A_n S^n \end{aligned} \qquad (11)$$

Each coefficient a_i is now multiplied by the scale factor f raised to the i power. Typical values for f that bring all coefficients within the allowable range of the computer are 1E+4 or 1E+5.

There is a very important bi-product of scaling in this manner. When performing response calculations the speed variable s is in radians per second. Suppose that the only speeds of interest are those that are less than 10,000 rad/s (or roughly 1500 Hz or 100,000 rpm). If we then set the scale factor f equal to 10,000, all the scaled speeds that we are interested in will have magnitudes less than one by virtue of equation (10).

Now direct attention to the coefficients A_i of the scaled polynomial. Consider any two coefficients A_i and A_j where j is greater than i. The sum of these two terms of the polynomial is

$$sum = A_i S^i + A_j S^j$$

Remember that j is always greater than i and for the problem at hand S will always be less than one. If A_i happens to be more than A_j then the second term will always be smaller than the first term. In fact, we can say that if

$$A_i >> A_j$$

then we can also say that

$$sum = A_i S^i$$

In this way we can selectively eliminate terms of the scaled polynomial that will always be insignificant so long as S is less than one. This procedure is referred to as dynamic condensation.

By neglecting insignificant terms, the degree of the polynomial is reduced significantly. The degree of the polynomials in $[T_{LL}]$ can typically be reduced by a factor of six or more. This results in tremendous savings of execution time both in calculating the coefficients and in finding the responses.

SPEED DEPENDENT BEARING PARAMETERS

The stiffness and damping properties of bearings are often functions of shaft speed. This is certainly true for any fluid film bearing, and also for high speed rolling element bearings. A useful aspect of the PTM method is that bearing parameters in equation (1) can be expressed as polynomial functions of speed (actually $i\omega$). These functions can be used directly in equation (7a) so that the system polynomial matrix $[T]$ will automatically provide the required speed dependent affect. This is significant because equation (7a) would otherwise need be recomputed for each speed increment, and the efficiency of the method would be reduced.

OPTIMIZED MATRIX MULTIPLICATION

Two important details will be discussed here which greatly influence the computational efficiency of the PTM method. Both details concern the numerous matrix multiplications which must be carried out in equation (7a). First, all matrices in equation (7a) are eight by eight. But due to the shaft end boundary conditions, only the lower left quarter of $[T]$ is required in equation (7b). The right four columns of T are not required, and need not be computed. The second item concerns how each successive station is multiplied onto T_{LL} and E_L. For example, multiplying one station onto $[T]$ would most logically be done as

$$\begin{bmatrix} T_{UL} & T_{UR} \\ T_{LL} & T_{LR} \end{bmatrix}_n = \begin{bmatrix} S_{UL} & S_{UR} \\ S_{LL} & S_{LR} \end{bmatrix}_n \begin{bmatrix} I & 0 \\ M_{LL} & M_{LR} \end{bmatrix}_n \begin{bmatrix} T_{UL} & T_{UR} \\ T_{LL} & T_{LR} \end{bmatrix}_{n-1} \qquad (12)$$

As mentioned above, $[T_{UR}]$ and $[T_{LR}]$ are not required. Expanding equation (12) then gives

$$[T_{UL}]_n = ([S_{UL}] + [S_{UR}][M_{LL}])_n [T_{UL}]_{n-1} + ([S_{UR}][M_{LR}])_n [T_{LL}]_{n-1} \qquad (13a)$$
$$[T_{LL}]_n = ([S_{LL}] + [S_{LR}][M_{LL}])_n [T_{UL}]_{n-1} + ([S_{LR}][M_{LR}])_n [T_{LL}]_{n-1} \qquad (13b)$$

The M matrices contain two degree polynomials, and the S matrices are zero degree. Equations (13) should be carefully reviewed and compared to the following alternate form:

$$\begin{bmatrix} T_{UL} & T_{UR} \\ T_{LL} & T_{LR} \end{bmatrix}_n = \begin{bmatrix} I & 0 \\ M_{LL} & M_{LR} \end{bmatrix}_n \begin{bmatrix} S_{UL} & S_{UR} \\ S_{LL} & S_{LR} \end{bmatrix}_{n-1} \begin{bmatrix} T_{UL} & T_{UR} \\ T_{LL} & T_{LR} \end{bmatrix}_{n-1} \qquad (14)$$

Expanding gives

$$[T_{UL}]_n = [S_{UL}]_{n-1}[T_{UL}]_{n-1} + [S_{UR}]_{n-1}[T_{LL}]_{n-1} \tag{15a}$$

$$\begin{aligned}[T_{LL}]_n = &([M_{LL}]_n[S_{UL}]_{n-1} + [M_{LR}]_n[S_{LL}]_{n-1})[T_{UL}]_{n-1} + \\ &([M_{LL}]_n[S_{UR}]_{n-1} + [M_{LR}]_n[S_{LR}]_{n-1})[T_{LL}]_{n-1}\end{aligned} \tag{15b}$$

It is readily seen that equations (15) require approximately one third fewer multiplications than equations (13). Equations (15) can also be programmed to use less computer memory for storage of intermediate results. This gain is worth the extra bookkeeping needed at stations 1 and N. It is interesting to note that equations (15) would themselves have been the logical first choice if the station definition of Figure 2 had the shaft to the left of the mass.

EXAMPLES

Two example cases will be presented to demonstrate how the PTM method for responses compares to both the CTM and FE methods. The CTM program is based on the work of Lund (1974) and does its calculations in double precision. The FE program uses single precision and is actually a modal synthesis program which operates on a truncated set of normal modes computed with a separate FE eigenanalysis code. The first example case will be a uniform shaft on two bearings. This model will be used to explore the performance of the methods for different model sizes, and different numbers operating speed increments. This model will also be used to show the effect of dynamic condensation on the speed and accuracy of PTM method. The second example case is of a multistage centrifugal compressor. This model will demonstrate how the PTM method performs on a more realistic analysis. All analyses were performed on a 25 Megahertz 486 class IBM-PC compatible desktop computer.

The first example rotor is a solid uniform shaft 1 inch in diameter, 20 inches long, with a bearing at each end having 5,000 lb/in stiffness and 5 lb-s/in damping. Rotary and gyroscopic inertias were included. One gram-inch of unbalance was applied at midspan and at one bearing, and the speed range for response analysis is from zero to about twice the first critical speed. In all cases the PTM and CTM programs gave virtually identical answers. The FE program also gave identical answers when all modes were retained, but the results in the figures to follow employed only the first 5 free-free shaft bending modes (enough to be within 1% on midspan response amplitude). Figure 3 shows execution times with the shaft divided into different numbers of beam elements. Figure 3 also shows the effect of changing the number of speed increments for analysis. The times shown in the figure for the FE program do not include the times for normal mode analysis. Also, the FE program could not run the 40 station case due to memory limits. Figure 3 shows the speed advantage of the PTM method. From the limited range of cases shown in the figure it is seen that the PTM method ranges from 3.5 times to 10 times faster than the CTM method, and from over 6 times to almost 30 times faster than the FE method. An important aspect seen in Figure 3 is that the speed advantage of the PTM method increases both while increasing the number of elements in the model, and also while increasing the number of speed increments for the analysis.

Figure 4 shows the difference that dynamic condensation makes for the PTM method. The speed gain due to condensation becomes greater with increasing number of stations, and diminishes slightly with increasing numbers of response frequencies. At the 5 station level, there is no gain with condensation because the system polynomials are simply not big enough (degree=2*6=12). At greater than 5 stations there is an increasing gain due to condensation. Figure 5 demonstrates the effectiveness of condensation at retaining full accuracy for the response analysis. Figure 5 shows that the cutoff in accuracy is somewhat dependant on the frequency of analysis with respect to the modes of the rotor. In Figure 5 "low speed response" is near the first bending mode of a rotor, and "high speed response" is above the third bending mode of a rotor. In practice, it is quite easy to reap the speed gain of polynomial condensation, while maintaining ample margin against inaccuracy.

The second example case is of a high performance multistage centrifugal compressor typical of the process industry, Figure 6. The rotor is 7 feet long, weighs 1077 lbs, is supported by tilting pad bearings, and operates supercritical at 12,000 rpm. Because of its back to back impeller arrangement, there is a high pressure honeycomb seal at midspan. The rotordynamic model has 55 stations and includes the predicted stiffness, damping and cross-coupling of the bearings and honeycomb seal (see Figure 6). Figure 7 shows partial results of an analysis using 256 speed increments. All three programs computed the same responses (the FE program required the first 5 free-free bending modes for 1% error). The execution times, in minutes, were; PTM=0.22, CTM=2.4, FE=28.5. The speed advantage of the PTM method thus makes it possible to run more design iterations, unbalance distributions cases, etc., than one could with the other methods.

CONCLUSIONS

The polynomial transfer matrix method is effective for performing rotordynamic eigenanalyis, and has now been successfully applied to unbalance response analysis. The method has been described in detail including the technique of automatic dynamic condensation and other optimizing procedures. Two example rotor models were used to demonstrate the accuracy and speed of the method. Responses computed via the polynomial method were found to match exactly those computed with the conventional transfer matrix method and the finite element method. The speed advantage of the polynomial method was found to range from 3.5 to 100 times faster than the other two methods depending on the particular case. This speed advantage is significant because a single run of unbalance response analysis of the type required by several of the API purchase specifications for new equipment will take only around 10 seconds with the polynomial method on a desktop computer, whereas 10 minutes or more may be required with the other methods.

REFERENCES

1. Holzer, 1921, *Die Berechnung der Drehschwingungen*, Springer-Verlag, Berlin.

2. Horner, G. C., and Pilkey, W. D., 1977, "The Riccati Transfer Matrix Method," ASME Paper No. 77-DET-32.

3. Lund, J. W., 1974, "Stability and Damped Critical Speeds of a Flexible Rotor in Fluid-Film Bearings," ASME *Journal of Engineering for Industry*, Vol. 96, No. 2, May.

4. Lund, J. W., and Orcutt, F. K., 1967, "Calculations and Experiments on the Unbalance Response of a Flexible Rotor," ASME *Journal of Engineering for Industry*, Vol. 89, No. 4, November.

5. Murphy, B. T., 1984, *Eigenvalues of Rotating Machinery*, Doctoral Thesis, Texas A&M University, May.

6. Murphy, B. T. and Vance, J. M., 1983, "An Improved Method for Calculating Critical Speeds and Rotordynamic Stability of Turbomachinery," ASME *Journal of Engineering for Power*, V. 105, July.

7. Myklestad, N. O., 1944, "A New Method of Calculating Natural Modes of Uncoupled Bending Vibration of Airplane Wings and Other Types of Beams," *Journal of Aeronautical Sciences*, Vol. 11, No. 2, April.

8. Prohl, M. A., 1945, "A General Method for Calculating Critical Speeds of Flexible Rotors," ASME *Journal of Applied Mechanics*, Vol. 12.

9. Vance, J. M., 1988, *Rotordynamics of Turbomachinery*, Wiley Interscience.

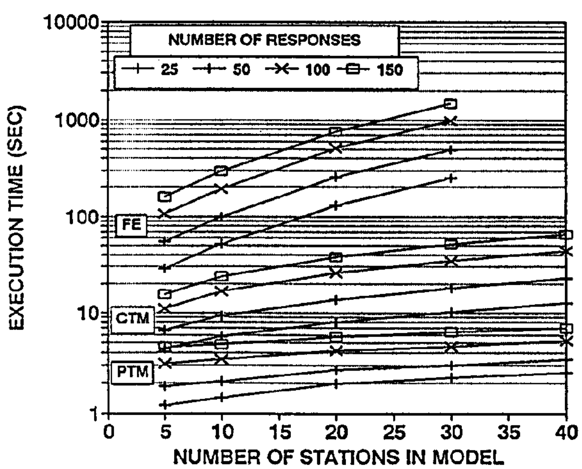

FIGURE 3. EXECUTION TIMES FOR THE PTM, CTM AND FE METHODS.

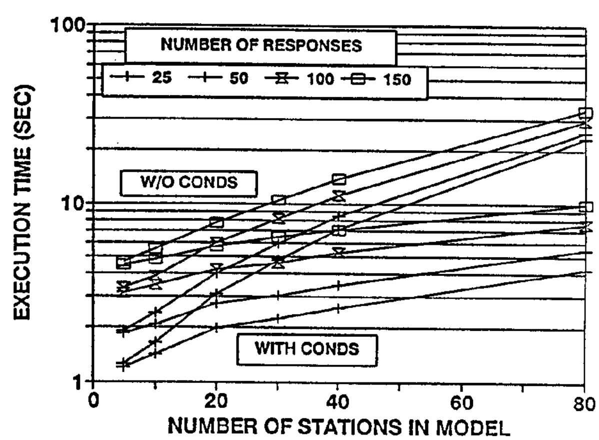

FIGURE 4. EFFECT OF CONDENSATION ON EXECUTION TIME FOR THE PTM METHOD.

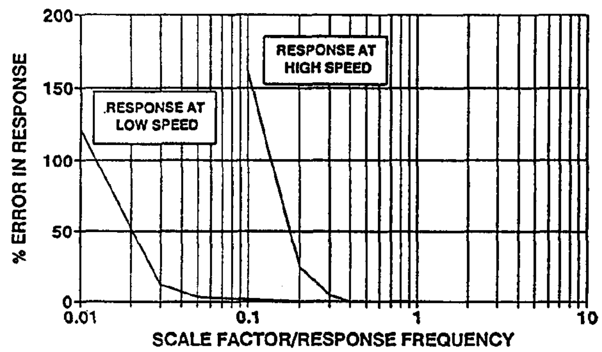

FIGURE 5. ERROR IN COMPUTED RESPONSE DUE TO CONDENSATION.

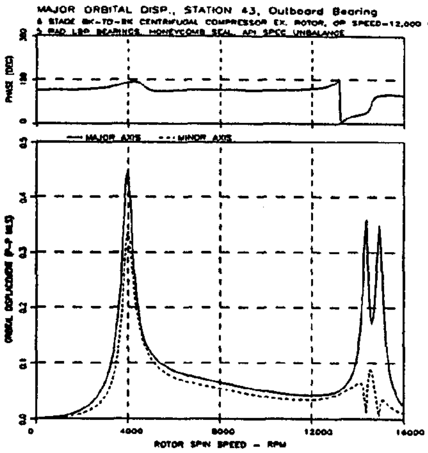

FIGURE 7. SAMPLE RESPONSE PLOT FOR COMPRESSOR ROTOR.

CENTRIFUGAL COMPRESSOR EXAMPLE ROTOR

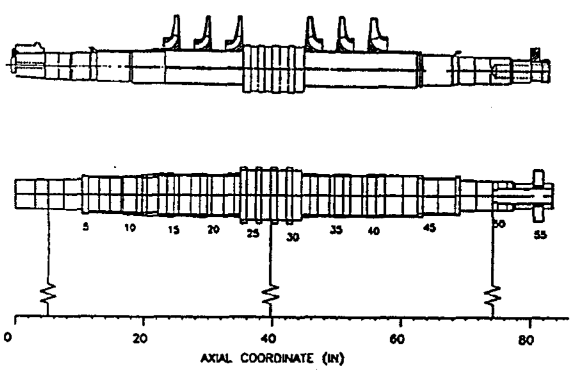

FIGURE 6. CENTRIFUGAL COMPRESSOR EXAMPLE ROTOR.